Problem 32.10

The solenoid has a length of .16 meters and 420 winds. The uniform rate of decrease of current (this is di/dt) is .421 amps per second. This induces an EMF of 175 microvolts. What's the solenoid's radius?

We need a relationship that links an inductor's induced EMF to its actual geometry. If you noticed there is a defining equation and example problem in the book that does it all for you (Equ. 32.4 in Example 32.1 in the version of the book I'm looking at), you'd be done. If you didn't notice that, you would have to do what I did and muddle your way through deriving your own expression.

As is always the case when muddling, you just kind of write down relationships that are true and hope you end up where you want to be.

To that end:

We know:

$$|\varepsilon_{induced}| = L\frac{di}{dt}$$

$$\Rightarrow \quad \frac{di}{dt} = \frac{|\varepsilon_{induced}|}{L} = \frac{N\frac{d\Phi_B}{dt}}{L} = \frac{N\frac{d(\vec{B}\bullet\vec{A})}{dt}}{L} = \frac{N\frac{d(BA\cos 0^\circ)}{dt}}{L}$$

Remembering that the magnetic field down the axis of a solenoid is:

$$B = \mu_o ni,$$

where "n" is the *number of winds per unit length*, we can write:

$$\frac{di}{dt} = \frac{N\frac{d(BA\cos 0^\circ)}{dt}}{L} = \frac{NA\frac{d(\mu_o ni)}{dt}}{L} = \frac{NA\mu_o n\frac{d(i)}{dt}}{L}$$

Calling the length of the solenoid "l" (this for the *number of turns per unit length* "n" term—see next page), noting that the cross-sectional area of the solenoid is "πr^2," canceling the "di/dt" terms on both side and rearranging, we get:

$$L = NA\mu_o n = NA\mu_o\left(\frac{N}{l}\right) = \frac{N^2\mu_o A}{l} = \frac{N^2\mu_o(\pi r^2)}{l}$$

With the inductance "L," we can proceed with our EMF expression and write:

$$|\varepsilon| = L\left(\frac{di}{dt}\right) = \left(\frac{N^2\mu_o(\pi r^2)}{l}\right)\left(\frac{di}{dt}\right)$$

$$\Rightarrow \quad (175x10^{-6}V) = \left(\frac{(420)^2(1.26x10^6)(\pi r^2)}{(.16\text{ m})}\right)(.421\text{ A/s})$$

$$\Rightarrow \quad r = 9.77x10^{-3}\text{ m}$$